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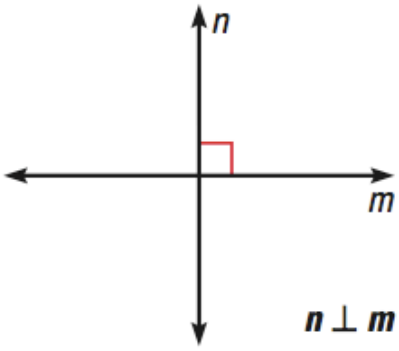
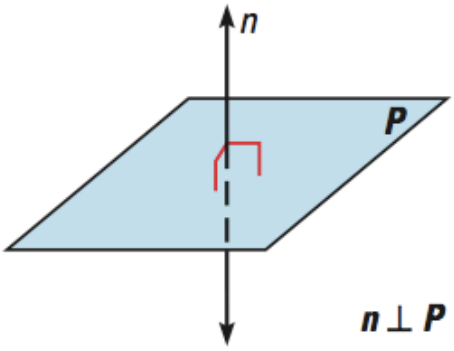
Definitions and Biconditional Statements (pp 79-81)

Two lines are called **perpendicular lines** if they intersect to form a right angle. A **line perpendicular to a plane** is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it. The symbol $\perp$ is read as "is perpendicular to."





All definitions can be interpreted "forward" and "backward." For instance, the definition of perpendicular lines means (1) if two lines are perpendicular, then they intersect to form a right angle, *and* (2) if two lines intersect to form a right angle, then they are perpendicular.

Conditional statements are not always written in if-then form. Another common form of a conditional statement is *only-if* form. Here is an example.

It is Saturday, only if **I am working at the restaurant**.

Hypothesis **Conclusion**

You can rewrite this conditional statement in if-then form as follows:

If **it is Saturday**, then **I am working at the restaurant**.

A **biconditional statement** is a statement that contains the phrase "if and only if." Writing a biconditional statement is equivalent to writing a conditional statement *and* its converse.

Mathematical Definition: All definitions must be biconditionals.

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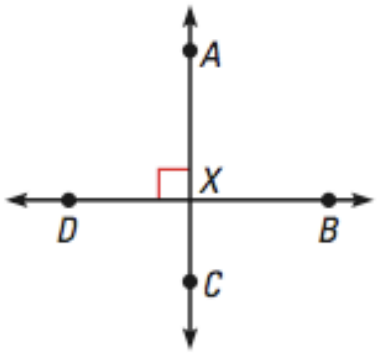
EXAMPLE 1 Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. Points D, X, and B are collinear.

b. $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{DB}$.

c. $\angle AXB$ is adjacent to $\angle CXD$.

SOLUTION

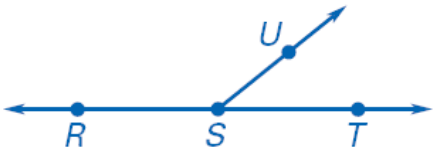
a. This statement is true. Two or more points are *collinear* if they lie on the same line. The points D, X, and B all lie on line $\overleftrightarrow{DB}$ so they are collinear.

b. This statement is true. The right angle symbol in the diagram indicates that the lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ intersect to form a right angle. So, the lines are perpendicular.

c. This statement is false. By definition, adjacent angles must share a common side. Because $\angle AXB$ and $\angle CXD$ do not share a common side, they are not adjacent.

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

1. Points R, S, and T are collinear.



2. $\overrightarrow{SU}$ is perpendicular to $\overrightarrow{RT}$.

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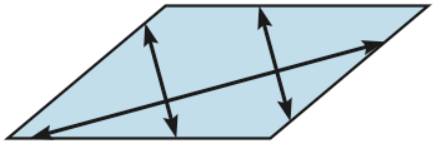
EXAMPLE 2 *Rewriting a Biconditional Statement*

The biconditional statement below can be rewritten as a conditional statement and its converse.



Three lines are coplanar if and only if they lie in the same plane.

Conditional statement: If three lines are coplanar, then they lie in the same plane.

Converse: If three lines lie in the same plane, then they are coplanar.

3. Rewrite the biconditional statement as a conditional statement and its converse.
Two lines intersect if and only if their intersection is exactly one point.

EXAMPLE 3 *Analyzing a Biconditional Statement*

Consider the following statement: $x = 3$ if and only if $x^2 = 9$.

a. Is this a biconditional statement?

b. Is the statement true?

SOLUTION

a. The statement is biconditional because it contains "if and only if."

b. The statement can be rewritten as the following statement and its converse.

Conditional statement: If $x = 3$, then $x^2 = 9$.

Converse: If $x^2 = 9$, then $x = 3$.

▶ The first of these statements is true, but the second is false.
So, the biconditional statement is false.

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4. Consider the following statement: $x^2 < 49$ if and only if x < 7.

a) Is this a biconditional statement?

b) Is the statement true?

Consider the following statement: $x^2 = 4x$ if and only if x = 4.

5. Is this a biconditional statement?

6. Is the statement true?

EXAMPLE 4 ***Writing a Biconditional Statement***

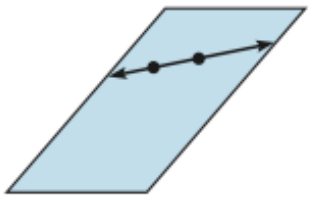
Each of the following statements is true. Write the converse of each statement and decide whether the converse is *true* or *false*. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

a. If two points lie in a plane, then the line containing them lies in the plane.

b. If a number ends in 0, then the number is divisible by 5.

SOLUTION

a. Converse: If a line containing two points lies in a plane, then the points lie in the plane.



The converse is true, as shown in the diagram. So, it can be combined with the original statement to form the true biconditional statement written below.

Biconditional statement: Two points lie in a plane if and only if the line containing them lies in the plane.

b. Converse: If a number is divisible by 5, then the number ends in 0.

The converse is false. As a counterexample, consider the number 15. It is divisible by 5, but it does not end in 0, as shown at the right.

$10 \div 5 = 2$
$\blacktriangleright 15 \div 5 = 3$
$20 \div 5 = 4$

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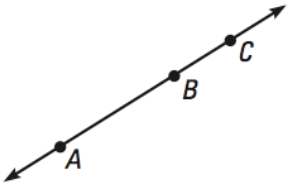
7. The following statement is true. Write the converse and decide whether it is true or false. If the converse is true, combine it with the original to form a biconditional. If $x^2 = 4$, then x = 2 or x = -2.

EXAMPLE 5 ***Writing a Postulate as a Biconditional***

The second part of the Segment Addition Postulate is the converse of the first part. Combine the statements to form a true biconditional statement.

SOLUTION

The first part of the Segment Addition Postulate can be written as follows:



If B lies between points A and C, then $AB + BC = AC$.

The converse of this is as follows:

If $AB + BC = AC$, then B lies between A and C.

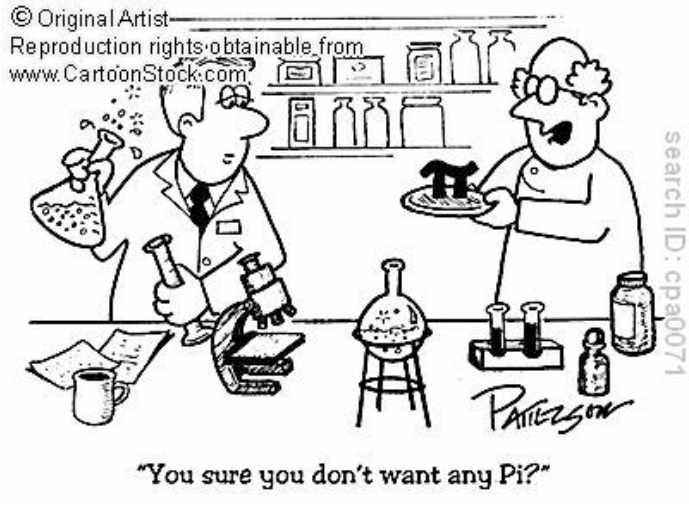
Combining these statements produces the following true biconditional statement:

Point B lies between points A and C if and only if $AB + BC = AC$.

STUDENT HELP

Study Tip

Unlike definitions, not all postulates can be written as true biconditional statements.



"You sure you don't want any Pi?"

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8. The converse of the Angle Addition Postulate is true. Write the converse and combine it with the postulate to form a true biconditional statement.

9. The following statement is true. Write the converse and decide whether it is true or false. If the converse is true, combine it with the original to form a biconditional. If two planes intersect, then they contain the same line.

10. _____ Which statement below can be combined with its converse to form a true biconditional statement?

A. If an angle measures $100°$, then the angle is obtuse.

B. If two angles are complementary, then the sum of their measures is $90°$.

C. If two angles are a linear pair, then they are supplementary.

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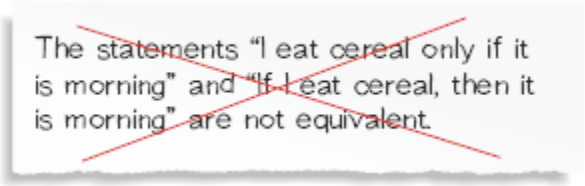
11. Describe in your own words what a true biconditional statement is.

12. What is wrong with Jared's argument below?



Tell whether the statement is a biconditional.

13. I will work after school only if I have the time.

14. An angle is called a right angle if and only if it measures 90°.

15. Two segments are congruent if and only if they have the same length.